EXPLORING CHAOS TO MODEL THE DESIGN PROCESS

A Thesis

by

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ABSTRACT

Exploring Chaos to Model the Design Process. (August 1990)

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As a first step in developing a general model which can ultimately monitor and suggest improvements throughout the evolution of a design, this research aims to construct a low-order mathematical model which describes the fundamental activities that drive the design process. The construction of a mathematical model for the design process required an initiation of a study aimed at an understanding of the most fundamental processes involved.

To that end, the first objective was to identify a set of variables which collectively describe the 'state' of a design. Next, a qualitative model that outlined the processes between the variables was developed. Analogies were drawn between elements of the qualitative model and those of dynamic systems. Based upon these analogies and the inherent properties of simple chaotic equations to depict complex behavior, a mathematical model was developed. The general dynamics of the model have been analyzed, and observed implications of chaos within the design process have formed the basis for proposed subsequent research.

ACKNOWLEDGEMENTS

I would like to thank Dr. David Jansson and the Institute for Innovation and Design in Engineering for supporting this research. I appreciate the consultation of Dr. Steven Smith. I look forward to working with him more closely in the next phase of the research in which the cognitive aspects of the design process will be investigated in more detail. I thank Dr. Chris Burger for his enthusiasm and support for this work. And to Dr. Sherif Noah. I extend my gratitude for keeping his door always open for me and for the endless time he spent teaching me about the world of nonlinear dynamics. I extend my appreciation to Sandra Lippka and Zuhair Arzouni for helping with the figures and reviewing this manuscript. And most of all, I thank God.

DEDICATION

I dedicate the efforts of this research to my loving mother.

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INTRODUCTION

Until recently, the design process has been treated as a black box. With the growing interest in substantially improving the way we design, it is becoming increasingly apparent that we must first achieve a good understanding of the design process and the fundamental activities that drive it. It may no longer be sufficient to consider good design practice as depending solely on intuition and talent. As long as the process remains poorly understood, little progress will be made in devising ways to design better.

Thus far, the most common approach taken in the literature, to the study of design is the generation of qualitative models that attempt to describe the design process. In general, two types of models were usually proposed: *descriptive* models and *prescriptive* models (1). As implied by their names, the former attempt to describe how we design while the latter prescribe methods or methodologies for improving the effectiveness and efficiency of the design process. Although such models may be useful, mathematical models will serve as more powerful tools for increasing the level of understanding of any dynamic process such as design. Even in disciplines in the socio-psychological fields, mathematical models are being developed to describe processes that were once studied only qualitatively. For example, Langs' models (2) describe certain psychoanalytic phenomena while other models have been developed for serial memory (3), dynamics of political coalitions (4), learning theories (5), and a wide variety of natural systems (6). These models attempt to provide an understanding of the intricate dynamics behind such processes. All of them with the exception of Langs' model are based upon probabilistic approaches which provide some predictive ability, but do not shed much light on the fundamental dynamics

This thesis follows the style and format of *American Scientist* behind the process of interest. In contrast, the most recent model by Langs (2) proposes a nonlinear dynamic model which exhibit chaotic behavior to describe some socio-psychological phenomena.

Similarly, design consists of a myriad of complex, interwoven processes. Attempts to identify every activity which may occur through the course of a design, and to understand how they influence each other may very well prove impossible. Even if it was possible, such a model would most likely be much too complex to yield any fruitful insights. However, if the fundamental activities that occur in a design process can be identified, it may be possibleto develop a simple, low-order dynamic model which concentrates on those activities that are most fundamental to the design process.

When attempting to mathematically model such a complex process as design, many challenges arise. Firstly, the process itself remains poorly understood. Moreover, many factors, which are numerous and not even fully identifed affect the outcome of a given design. Finally, the design process is not as deterministic as a 'step by step recipe' nor is it a totally random process. Although the outcome and the path taken to reach that outcome may not be deterministic, the *nature* of the process is. A candidate emerges on which modelling a design process can be based, namely, a nonlinear deterministic model capable of exhibiting chaotic behavior (7).

Consider some of the similarities between the design process and chaotic behavior (Appendix A contains a brief overview of nonlinear dynamics theory and chaos terminlogy).

THE CHAOTIC NATURE OF THE DESIGN PROCESS

In design, it is obvious that a slight change in the definition of the problem to be solved may lead to dramatically different solutions. For example, if two designers identify the same need from different perspectives and thus define the problem differently, each design will almost certainly evolve differently. Also, for the same designer and design problem, a small change in insight derived from new information can lead a designer to a completely different solution. It is clear that the outcome of a design process is not only *sensitive to initial and current conditions* but also to any abrupt changes that occur within the process. This type of behavior is a fundamental property of chaotic nonlinear systems (8). Sensitivity to initial conditions in chaotic systems is recognized to be due to 'bounded divergence' or 'stretching and folding' which causes an infinitesimal perturbation in the state of the system to grow exponentially and eventually leads to a completely different state than the one the system would have reached had it not been perturbed.

Also, the dynamics within certain 'attractors' (8) of a nonlinear dissipative system resemble certain phenomena occuring within the design process. One such attractor could, for example, be the case of design fixation (9), in which a design has settled into an undesirable regular or " fixed state". Another attractor occurs when the design process reaches some final solution which undergoes little or no change upon further iterations. Once the process has reached the neighborhood of either of these two states (fixation or solution), it is 'attracted' to that state. Therefore, design fixations as well as solutions which satisfy all of the design requirements resemble fixed point attractors in nonlinear dynamic systems.

Once a designer is fixated, some abrupt action must be taken to get out of it. This action may either be in the form of a decision to approach the proplem from a different perspective or to seek new information which may provide some insight into the problem. Similarly, when a dynamic system is within an attractor, it will remain there unless some change is made to the system (by varying the parameters or externally displacing the system sufficiently).

Another similarity between design and chaos is the feature of "self-similarity". The process of generating solutions to a problem is identical at all stages of a design, from the concept levels to the detailed levels. For instance, the process of identifying a problem, analyzing it, creating a solution, and evaluating it are common to all levels of the design, that is, they are self similar, whether we are solving the whole problem or just one aspect of the problem. This can also be shown from the argument that the details of one designer's problem may be the whole of another designer's problem. Therefore, the processes operating on one level of design are similar at all other levels of design. This is the same as the self similar (fractal) behavior of chaotic systems as illustrated by the example of the Henon map (10). This is shown in figure 1 which maps the iteration of the form:

$$x_{i+1} = 1 - a x_i^2 + y_i$$

$$y_{i+1} = b x_i$$

The top graph in figure 1 shows the entire range of points generated from the iterative mapping. The attractor appears to exhibit a three leaf structure. If the section enclosed in the window is magnified (bottom-left graph), we find that the upper most leaf is composed of three leaves of identical structure to that drawn at the previous scale. If the top leaf is magnified once more, it is also seen to consist of the same structure as the previous scales. Futher magnification will continue to reveal that the same order exhibited by the larger scales is similar to that of the smaller scales ad infinitum.

Consider yet another common characteristic between the design process and chaotic behavior. Design is a continually converging-diverging process. For example, when a designer chooses and develops a concept, he is converging upon a solution. In other words, the design begins to settle onto one state. On the other hand, when a designer scans for concepts and thinks on an abstract level as is the intent of productive brainstorming, the design process is diverging. The design process does not settle onto a single state but rather jumps from state to state as each idea may lead to new and different ideas. Developing single concepts further, sometimes referred to as *vertical thinking* is a convergent process while the generation of alternative

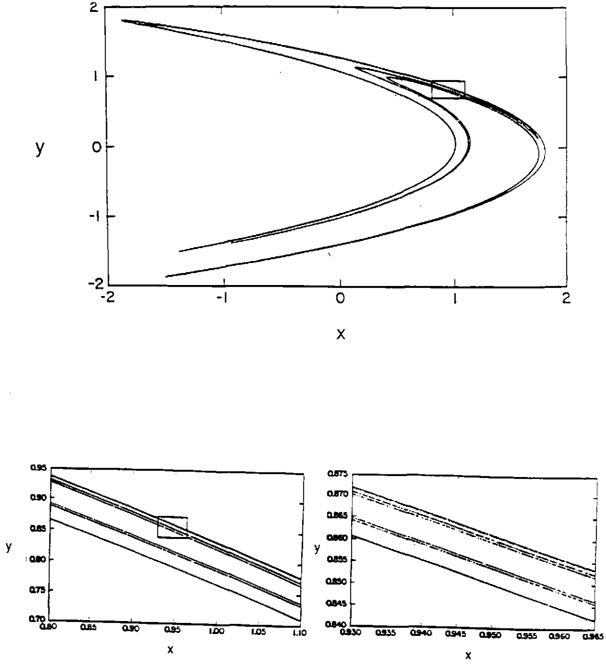


Figure 1. The self-similarity exhibited by the Henon Map. The same 'three leaf' structure exists at all levels of magnification.

solutions, *lateral thinking* is a diverging process. In this sense, design, represented as a continually converging-diverging process, resembles a chaotic system. Now consider two nearby points depicting different initial states of a stable, chaotic dynamic system. The trajectory of each point represents the evolution of the system from that initial state. With time, the two trajectories exponentially diverge from each other. However, since the system is stable, the trajectories must remain within a bounded space. Therefore, they must reconverge toward each other. This behavior is characterized by a series of, what is termed in modern dynamic theory as, 'stretching and folding' resulting in sensitivity to initial conditions. In this sense, design represented as a continually diverging-converging process resembles a chaotic system.

Some of the general characteristics common to both a chaotic system and the design process are listed in table 1. All of the similarities between design and chaos strongly suggest that the design process can be modeled as a chaotic system.

OBJECTIVES

The main objective of this research is to explore the ability of chaotic nonlinear dynamics to model the fundamental activities comprising the design process. In particular, the objectives are as follows:

1. Identify the essential underlying processes common to all levels (conceptual to detailed) of the design process.

2. Identify a set of variables that collectively depict the state of a design.

3. Develop a qualitative model which outlines the relationships between the state variables.

4. Explore the feasibility of using chaotic equations to model the processes between the state variables and how they depict the evolution of a design.

Table 1. Common characteristics between design and chaotic systems.

Chaotic Systems	The Design Process
Chaotic equations are highly sensitive to initial conditions.	The outcome of a design may vary widely with the way a design problem is formulated from the need.
Chaotic systems exhibit self-similar (fractal) structure.	The characteristics of the design process are common to all levels (conceptual to detailed) of design.
Trajectories depicting the time evolution of a chaotic system continually converge and diverge in relation to each other.	Design can be viewed as a series of converging-diverging activities.
Attractors represent steady state solutions to the governing equations of a dynamic system.	Solutions to design problems are analogous to 'steady state' conditions that satisfy constraints posed by design problems.
An addition of energy to a chaotic system may drive it to new attractors.	Any new information introduced to the design process may lead to different solutions.

CHAPTER II

THE FUNDAMENTAL PROCESSES OF DESIGN

As was stated in the introduction, the first objective of this research was to identify some of the fundamental activities of the design process. In order to accomplish this task, the following approach was taken:

1. The literature was surveyed for descriptive models of the design process.

2. Models that attempted to fathom the underlying processes in design were chosen for further study.

3. These models were then used to develop the basis for the mathematical model in this study.

A survey of the design literature reveals that much of the work done in early studies of the design process revolves around the generation of methods and rules which lead toward better design. Even today, systematic approaches for the solution of design problems are continually being sought to aid the designer. Many of these approaches lead to the development of *design methods*, which as Cross (11) points out, aim to 'formalize' aspects of the design process and 'externalize' the designer's thoughts. Formalization prevents missing of simple and fundamental ways to solve a particular design problem. Externalization attempts to bring out the designer's thoughts onto paper through sketches, diagrams, and charts to facilitate and stimulate further development.

It may be difficult to define exactly what a 'good design' is. For the purpose of this study, consider a good design as a solution which satisfies all of the design requirements. Hence, optimization is not in the scope of this research.

Through a suggested set of rules, French (12) discusses some general strategies to help the designer improve the way he formulates solutions to design problems. While his suggested schemes are of great value in improving one's approach to generating design alternatives, it, like most other design schemes, says little about the design *process* itself. Although design methods are useful in specific situations, they do not meet the need for a more global understanding of the design process.

Other efforts in past studies of the design process have focused on developing two types of qualitative models: *descriptive* models and *prescriptive* models (1). As implied by their names, the former attempt to identify and describe how we design while the latter prescribe a method or a methodology to improve the effectiveness and the efficiency of the way in which we design.

These models often present highly deterministic block-diagrammed schemes which impose a heirarchal structure to design (13). Only a few models attempt to portray the underlying nature of the design process. In other words, little effort has been made to identify and depict the fundamental activities which comprise design and how these activities interact.

Two examples of models that attempt to fathom the underlying processes in design are March's model (14) and Jansson's model (15).

MARCH'S MODEL

March (14) proposes that the design process is simply the cyclic iteration among three logical processes: production, deduction, and induction (figure 2). He bases his model upon Peirce's work (16) in which three types of logical reasoning were identified. *Deduction* (analytic reasoning), is the definite conclusion of a specific result from a general rule. Additionaly, there are two types of synthetic reasoning: *induction*, the inference of a general rule from a specific case, and *abduction*, the creation of a specific result from a general rule and another specific result. Abduction, or as March refers to it, production, is the only logical process which proposes new information. Deduction and induction simply manipulate existing information.

Based upon this hypothesis, March proposes that the creation of a solution is a productive process since some new information is created based upon existing knowledge (description of a solution). Next the solution is analyzed and its performance is predicted. This is a deducive process since the designer uses factual knowledge and fundamental physical principles to assess the performance of the solution. Finally the solution is evaluated. Evaluation is an inductive process since generalizations are inferred from the analysis of a specific solution.

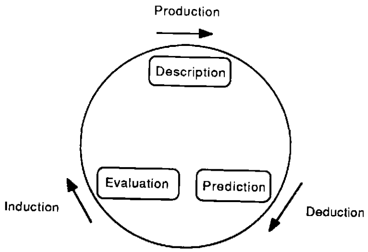


Figure 2. March's depiction of design as a sequence of three rational processes (14)

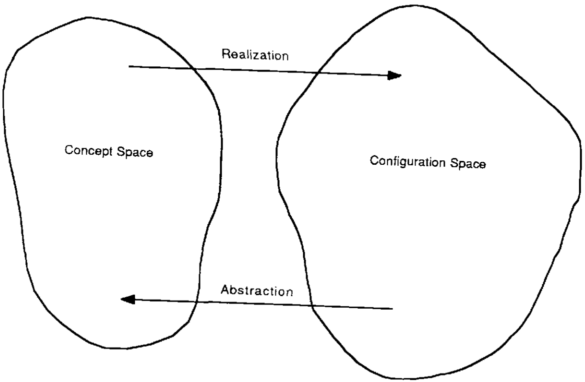


Figure 3. Jansson's two space model (15)

JANSSON'S MODEL

Jansson (15), on the other hand takes a more fundamental approach. He identifies two separate spaces which host the activities that occur in engineering design (figure 3). According to this model, at any given time, the conceptual design process can be modelled cognitively as taking place in one of two spaces: concept space and configuration space. Within concept space, general ideas are formulated. Notions developed in concept space are mostly based upon fundamental principles where the thinking is abstract. Here the designer defines the problem, analyzes it, and attempts to determine 'the natural laws' which may lay the foundation to a good solution. Concepts then act as the basis for physical solutions which are in configuration space.

In configuration space, the designer attempts to generate a solution which satisfies the design requirements from the concept. Upon evaluation of the solution, the designer identifies new problems. By moving back to concept space, he *generalizes* upon the knowledge he gains from evaluating a specific solution. Again, he seeks general concepts to solve the newly defined problems.

CONSTRAINT-DRIVEN MODEL

Another model which depicts design from a different perspective examines the nature of constraints and the impelling role they play within the design process (17). This model focuses on how constraints govern the application of a concept into an actual solution. In other words, this model describes the process which may occur in Jansson's configuration space.

When a designer has generated and chosen some concept on which to base his design, the total set of constraints plays a strong role in shaping the application of the concept into a physical solution. It is actually the set of constraints which creates the framework of a solution since any design which satisifies the constraints is an acceptable solution. Therefore, creation of a physical solution from a raw concept is a *constraint-driven* process.

In general, consider the total set of constraints that may be imposed on the solution of a design problem. Obviously, all 'functional requirements', i.e. the objectives or tasks that the design solution needs to accomplish must be satisfied. These functional requirements can be derived from the need. For example, in the design of the internal support structures for aircraft horizontal stabilizers (17), the functional requirement is to transfer the shear forces imposed by the pressure loading from one outer skin of the stabilizer to the other.

Other constraints imposed upon the solution may pertain to the fabrication of the substructure. These would be denoted 'production requirements'. Common examples of production requirements are repeatability and production rate. Here repeatability is the ability to maintain consistent quality from part to part, i.e. it assesses the manufacturability of a given product with respect to maintaining fairly consistent quality. It is in some way a reflection of the ability to meet tolerances, or more specifically the ratio of required tolerances on a product's physical specifications to the tolerances capable of being achieved by the process. Production rate, on the other hand, is a measure of the time required to produce one part. This constraint reflects the complexity of the design by representing the minimum time needed for all the serial processes required to produce one part.

Other constraints which do not fit under the categories of, functional requirements and production requirements, may be classified under the general heading of 'specifications'. In the case of the support structure for the horizontal stabilizer, these may include geometrical and cost constraints. Geometric constraints may be all of the lower and upper bounds placed on size and shape. For instance, the support structure may have to fit within an airfoil shape. Cost does not only represent monetary or financially related constraints, but also represents any ojective function being minimized such as weight, size, or energy consumption.

All of the constraints discussed thus far must be satisfied in order for any design to be a solution. Essentially, they are fixed throughout the entire design process, i.e. these constraints are constantly being imposed upon the solution. Collectively, the total set of constraints (functional requirements, production requirements, and specifications) will be be classified as *fixed constraints*. They are depicted on the top line of figure 4.

The *creation* of a physical solution, i.e. the product and the method of producing it, from a raw concept is a constraint-driven process. In figure 4, the arrows from the fixed constraints point to a set of variables which describe the different aspects of the currently proposed solution. These will be denoted as *output variables*. In the typical

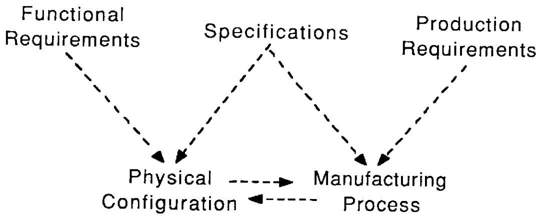


Figure 4. The different types of constraints within the design process. The bottom line consists of a typical set of output variables that describe the current form of the design. The origin of any arrow is the source of a constraint while the head of the arrow points to the output variable being constrained. The fixed constraints are depicted on the top line and the varying constraints act between the output variables.

case as illustrated in figure 4, the two output variables are *physical configuration* and *manufacturing process*.

The variable, physical configuration, contains information regarding the physical form of the solution. This is not to be confused with the shallower meaning of mere shape. Instead, configuration is the result of the concept operated on by the total set of constraints as to form some applicable solution. The manufacturing process is another output variable: it holds information regarding the method of fabricating the physical configuration. Unfortunately, this variable is often overlooked until the configuration is finalized. Since a configuration that is not manufacturable within the constraints of the production requirements is not a solution, considering the manufacturing process only downstream from the design process often leads to a 'patched up' or poor design. Therefore, it is important not to postpone manufacturing considerations till the basic design is completed but rather to include them upstream during the conceptual stages in an integrated manner as depicted in this model.

The model in figure 4 shows that the output variables are not only operated on by fixed constraints but also impose constraints upon each other. These constraints result from coupling between output variables which, in contrast to fixed constraints (which do not change through the evolution of a design), are subject to change as the associated output variables change. Therefore, they are denoted *varying constraints*.

For example, although a certain physical configuration may satisfy all of the fixed constraints, it may not be an acceptable solution if it is not manufacturable by the specified manufacturing process. In such a case, the manufacturing process is imposing a constraint upon the physical configuration. One of two actions may be taken: either another physical configuration is sought or another manufacturing process is sought. If the manufacturing process is changed, the constraints that were imposed upon the physical configuration will also be changed. If a different manufacturing process capable of fabricating the configuration is sought, the constraints that were once imposed upon the configuration may be lifted entirely.

In the constraint-driven scheme, two types of constraints acting within the design process were identified: fixed (external) constraints, and varying (coupling) constraints. Unlike other schemes, the three models discussed in this chapter (March's model, Jansson's model, and the constraint-driven model) attempted to identify the fundamental activities that drive the design process. Because of their ability to 'strip away' the complex facade and reveal different sides to the backbone of the design process, they were chosen to develop the basis for the mathematical model.

CHAPTER III

STATE REPRESENTATION OF DESIGN

Ultimately, a mathematical model is sought which will treat the design process as a dynamic system. Therefore the manner in which the state of a design at any given time is represented must be determined before quantification is possible. This constitutes the first objective of this chapter. A skeleton of the mathematical model will then be introduced which will lay the foundation for the actual equations developed in the proceeding chapter.

The state of a dynamic system is represented through a collection of variables which describe the time evolution of the system. For example, in cases involving mechanical systems, the variables depict the position and velocity associated with each degree of freedom of the system. Similarly, for the design process, a set of variables depicting the time evolution of a design must be identified. Collectively, they will represent 'state of a design'.

Firstly, consider the output variables discussed in the constraint driven scheme in chapter 2. Figure 4 illustrates the case in which the solution output variables were physical configuration and manufacturing process. In general, other variables may be required to represent the solution. For instance, when a manufacturing engineer is to design a process to fabricate a certain configuration, the manufacturing process may be the only variable, while the physical configuration would be a fixed constraint. In another situation, manufacturing aspects may not be part of the required solution at all. For example, in the design of some control system, the output variables may be the control logic and the physical configuration (the hardware which executes the logic).

As different design alternatives are generated, the current solution represented by the output variables may be replaced. Throughout the design process, the output variables partially depict the evolution of the design. We can conclude that the output variables of the constraint-driven scheme are a subset of the 'state variables' for the design process.

Thus far, the constraint-driven scheme has helped to identify one subset of the state variables. In chapter two, it was mentioned that the constraint-driven scheme can be considered as the process occuring within Jansson's configuration space. Therefore, we have only considered the process by which a concept is applied to form a proposed solution. Consequently, the state variables that were identified thus far are associated with the *application of a concept*. Now consider the source of a concept, i.e. *generation of a concept*. This is the process occuring in Jansson's concept space.

Unlike the process of applying a concept, constraints do not play a major role in the generation of a concept. As a matter of fact, the only direct limitation on this process may be the designer's own ability to utilize the resources available to him and to create alternative solutions. Therefore, the process of generating a concept is not a constraint-driven process as is the process of applying a concept. Instead, the generation of a concept is more a cognitive process based on information processing and use of knowledge. Consider figure 5 which could represent the main processes that may occur in Jansson's concept space.

Specific Knowledge denotes the amount of knowledge a designer has related to the nature of the problem he is attempting to solve. For example, a designer with a strong thermo/fluids background is able to draw upon more knowledge when solving a problem in that field than would an electrical engineer. From this definition, specific knowledge is a variable since as the problem is transformed (the current problem changes), the designer's knowledge about the current problem also changes. When solving a problem, the designer also draws upon his *general knowledge*. Unlike specific knowledge, general knowledge does not depend on the nature of the problem being solved. Rather, it is a measure of the culmination of the designer's acquired

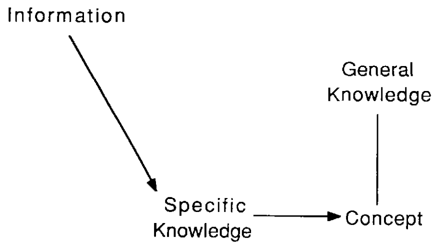


Figure 5. The major processes occuring in Jansson's concept space. specific knowledge and concept are variables, general knowledge is a parameter, and information is an external input into the design process. Arrows represent the general flow between the variables. Since general knowledge is a parameter, there is no arrow between it and the variable, concept.

information and experience. Therefore, general knowledge can be assumed not to vary throughout the design process.

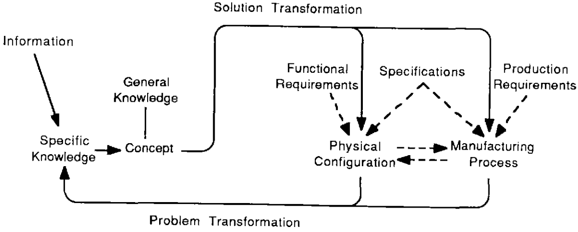
To clarify the difference between the two types of knowledge, consider general knowledge as the source of general ideas at the beginning of a designer's search for possible solutions. As the current problem being solved becomes more sharply defined, specific knowledge is required to develop the general ideas further. In problem solving, the generation of alternative solutions from a single idea is sometimes referred to as *lateral thinking* while the the devlopment of that idea is referred to as *vertical thinking*. Within such a context, general knowledge may be considered the source of lateral thinking and specific knowledge the source of vertical thinking. Since it does not vary throughout a design process, general knowledge is not a variable but rather a parameter.

If the design process is treated as evolving within some control volume which represents the designer, *information* is an external input. Essentially, information becomes the communication between the design process and the available resouces of the surrounding environment. Since it does not depend on any other parameter or variable, it is treated as an input to the specific knowledge of the designer

A *concept* is an abstract term and may have various connotations. Here it is defined as the set of fundamental principles upon which to base a solution. The concept is another variable since it can change throughout the evolution of a design. Concept and specific knowledge are the state variables within concept space. Now, the 'state of a design' can be defined as the set of state variables which collectively describe the design process, including the proposed solution at any point within the evolution of a design. The 'state of a design' can therefore be represented by the vector:

$$State\ of\ a\ Design = \begin{pmatrix} specific\ knowledge \\ concept \\ output\ variable\ 1 \\ \vdots \\ output\ variable\ n \end{pmatrix}$$

From figures 4 and 5, the skeleton of the mathematical model can now be constructed (figure 6). Through a simple example, the processes linking the generation of a concept and its application will be illustrated.



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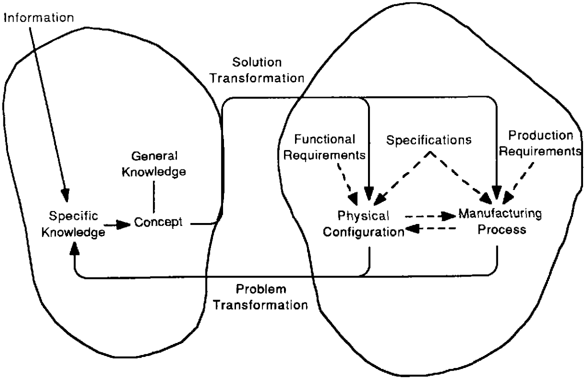

b Concept Space Configuration Space

Figure 6. Figure 6a depicts the skeleton upon which a mathematical model will be developed. Below in figure 6b, Jansson's two space model is overlayed on the mathematical model skeleton suggesting the dominant variables and processes which may operate in each space.

AN EXAMPLE TO ILLUSTRATE THE INTERACTION BETWEEN THE STATE VARIABLES

Consider the simple example of a need for some device to retard a continuously rotating shaft. Thus, the functional requirement of the device is to maintain and control the speed of the shaft. Further, a set of other requirements is generated. For instance, the minimum torque and speed of the system may be specified. In addition, the entire system must fit within a certain geometric envelope restricting volume and shape. A minimum production rate and life is required and a set limit may be placed on the total cost of producing the system. This combination of specifications and requirements collectively form the fixed constraints.

No doubt, much research can be conducted on how designers formulate, or how they *should* formulate, a problem from a need. For example, although it can be argued that the initial activity of analyzing the need to formulate a problem is carried out within concept space, the aim of the need analysis is to ideally identify and assign values of the constant constraints in configuration space. Once the constraints and ouptut variables pertaining to the overall problem within configuration space are defined, the conceptual design process starts.

For the example of the rotary shaft, our initial task may be to create a braking system for the shaft. From the designer's knowledge and information available to him, a first concept may be to slow the shaft mechanically through friction. This concept must now be transformed into a physical solution, perhaps a disk-caliper system. A casting manufacturing process which satisfies the production rate and cost constraints is chosen for the the disk-caliper configuration. The solution is now evaluated. Suppose the disk-caliper system does not satisfy the life requirements assuming high wear rates are involved. From the evaluation process, reduction of wear then becomes a new critical parameter. The information derived from evaluation has then been used to transform the problem and the process shifts back to concept space. After some analysis of the new problem, the idea of eliminating wear altogether may arise. This may lead to the concept of slowing the system through energy removal. In this sense, we have broadened the scope of solution alternatives over only those that dissipate energy through heat. Suppose that from this concept a flywheel is proposed to slow down the shaft by removing energy from it. Through the application of the concept to a proposed solution, the process jumps to configuration space. Due to the nature of the proposed flywheel geometry, the current manufacturing process, casting, may not be appropriate for producing the parts. In other words, when the new solution was evaluated, the constraints posed by the current manufacturing process were not satisfied by the new configuration. However, since the manufacturing process is a variable and not a fixed constraint, it may be changed. Now this information is used to transform the problem once more, this time, manufacturing being the critical variable. And again, the process returns to concept space. The iteration between concept space and configuration space continues.

DESIGN MODEL SKELETON

The general skeleton of the developed model for the design process lends some insightful perspectives on the process. The main fundamental activities which drive the design process have been identified. From these, we can determine general characteristics of the design process and how they can be modeled by dynamic systems. Some functional relationships can then be drawn relating the state variables, based upon analogies between design and dynamic systems, and the general structure of the model skeleton. Consider now the prominent observations pointed out by the model skeleton.

It is rather clear that the constraints play a dominant role in the design process. In general, the constraints define the *bounds* of the solution. The tighter the constraints, the fewer are the solutions which can satisfy them. It is therefore always desirable to define the problem such that there are as few constraints as possible. The constraints also drive the transformations between concept and configuration space. By imposing certain requirements on the solution's performance, geometry, and method of manufacture, they guide the transformation of concepts to physical configurations. And through evaluation, the unsatisfied constraints act as the basis for transforming the problem which may lead to an entirely different concept, and consequently increase the chances of an innovative solution.

As a prelude to the next chapter, consider the roles of parameters (constraints) and inputs (information) to the skeleton of the mathematical model. The process leading to a sound design may be viewed as a series of divergenent and convergent activities. The search for alternative conceptual solutions is a divergent activity. Thus creativity is often described as the ability to abandon focus and approach a solution from a very wide perspective in order to avoid fixation on a single concept. This is the prime motive of brainstorming . However, convergence is also necessary in order to develop any particular solution. This perspective of the design process is also supported by the proposed model skeleton. Information-fed concept generation acts as the divergent processes. The transformation from configuration space to concept space represents a divergence from specificity to abstract generality. Likewise, the opposing transformation represents convergence, a move from a general concept to a specific applied solution. The tighter or larger the number of constraints, the more bounded the solution. This is supported by the argument that when initially defining the problem or the need, one should identify only the important constraints so as to maximize the range of possible solutions. On the other hand, the more information available to the designer, the more principles and ideas the designer can pursue upon which to base a solution.

CHAPTER IV

THE MODEL

In this chapter, a first development of a model for the design process is proposed. Following, is a discussion of the reasoning behind the development of the mathematical model. The model is of low order since it attempts to represent only the most fundamental dynamics of design. The model was chosen to be a discrete, iterated mapping (see Appendix A) as opposed to a set of continuous differential equations for the following reasons:

1. The evolution of a design does not necessarily depend on time, rather on actions or sequences of actions.

2. It is more difficult to imagine that as a design evolves from one state to another, it passes through an infinite number of states (purely continuous process). Rather, the evolution of a design is more easily described through a set of discrete actions.

The final form of the proposed model is as follows:

$$c_{i+1} = \lambda(1 - d - x_i)(c_i + \theta_i \ sin\phi)$$

$$x_{i+1} = c_{i+1}(1 - d(1 - c_{i+1}) + \theta_i)$$

$$\theta_{i+1} = x_{i+1}(1 - c_{i+1})$$

where the **variables** (with subscript i denoting the current value of a variable) are:

c_i - Goodness of concept (ability to generate a good solution)

x_i - Goodness of solution

θ_i - Gain of Insight from evaluation

and the **parameters** are:

λ - Designer's Learning Ability

ρ - Designer's breadth of knowledge

d - Degree of total constraints

Other symbols used in the course of the development of the model which appear in the text of this chapter are:

Variables:

k_i - Specific knowledge

u_i - Goodness of physical configuration

v_i - Goodness of manufacturing process

α_i - Degree of problem transformation

Parameters:

d_1 - Degree of constraint on u_i from specifications

d_2 - Degree of constraint on v_i from specifications

b_1 - Degree of coupling: Constraining effect of u_i on v_i

b_2 - Degree of coupling: Constraining effect of v_i on u_i

All variables range from 0 to 1. All parameters range from 0 to 1 with the exception of λ whose lower bound is 0 and whose upper bound depends on the value of the other parameters. This will be discussed in the following chapter.

DYNAMIC CHARACTERISTICS OF THE SKELETON ELEMENTS

After constructing the skeleton of the model, certain dynamic characteristics of the elements (variables and parameters) emerge. The next step is to identify these dynamic characteristics by examining each of the elements and the nature of its role in the model. The first model element to be considered is information.

1. **Information** - As was discussed in the previous chapter, information is an external input to the design process. It is not limited to any designer. Obviously, information is a crucial element of the design process. Through available resources, the designer learns more about the problem and possible solutions. More importantly, through the exchange of information, the designer may gain some insight to the problem which may lead to some innovative solution. For instance, through a discussion with a colleague, the designer may see the problem from a completely different perspective. More subtle still, by watching out a window, some unrelated, natural activity may catch the designer's eye and trigger an insight into his design problem.

   Information is the sole communication window between the design process and the environment. Since the flow of information from the environment to the designer is, for the most part, independent of the remainder of the design process, it does not vary with each iteration with any deterministic fashion and is therefore not a *variable*. Rather, it is more of an external impulse on the design process. Within the context of the model, information indirectly affects each design state variable by *directly affecting specific knowledge*. Information can be modeled as a sudden force which acts on specific knowledge. In the text of the following discussion, the term 'knowledge' encompasses specific as well as general knowledge.

2. **Specific Knowledge** - From the model skeleton, there are two inputs into specific knowledge. In other words, there are two sources of knowledge gain: through information and through evaluation. Based on evaluation of a proposed solution, those constraints which are not satisfied come into focus and consequently drive the redefinition of the current problem (problem transformation). Many times, the *amount* of knowledge gained through such a process may not be significant, yet it may enable us to see the problem in a completely different way. The same is true with the gain of knowledge through available information. Although very little gain in the amount of knowledge may not lead to the generation of new solutions, a small transformation in the perspective by which the problem is viewed may yield a host of ideas. On the other hand, if a large amount of knowledge is gained about one particular means of solving the problem, it may tend to block the consideration of other means to solve the problem.

Therefore, it may be more accurate to model knowledge with two terms: the amount or depth of specific knowledge (represented as k) and the diversity or breadth of overall knowledge (represented as ϕ). The broader the knowledge of a designer, whether it be in a particular field or in general, the larger the pool of information the designer can draw upon to generate ideas to solve a given problem. Moreover, the more extensive the general knowledge of the designer, the greater the ability he has to think laterally and draw analogies between different processes. A 'broader picture' will enhance the ability of the designer to create innovative solutions. Depth of knowledge alone may not lead to the ability to 'scan' for solutions. In other words, a deep but narrow knowledge (a designer who is very knowlegable in one area but whose general engineering knowledge is limited), may lead only to the formulation of solutions based upon very similar concepts. Such a designer may be easily 'fixated' on one type of solution and lack the ability to diversify. A designer with a high breadth of knowledge may be more productive in a brainstorming session, for

Table 2. General effects of *breadth* (ϕ) and *depth* (k) of knowledge on concept formulation.

	Low k (shallow knowledge)	**High k** (deep knowledge)
Low ϕ (narrow field) (of knowledge)	Uninteresting: small ability to generate concepts.	Convergence: focusing upon one solution - positive if towards end of design - fixation if early in design process
High ϕ (broad field) (of knowledge)	Divergence wide but shallow - can generate ideas but unable to develop fully	High Potential wide and deep - abiltity to generate and develop ideas

example. Therefore, as summarized in table 2. the combination of different levels of depth and breadth of knowledge affect the general ability of a designer to generate conceptual solutions.

3. **Goodness of Concept** - The word 'concept' is itself a vaguely defined term and may project many different connotations. Here. a concept represents an idea or a principle upon which a solution may be based. A concept is therefore a raw solution not yet applied to the specific design problem. The skill of a designer in applying a concept to a specific problem will ultimately determine how good a solution is, that is, how well the solution satisfies the constraints. The concept acts as a *potential* for a solution. Consequently, it can be modeled as a potential energy term.

   In table 2. the ability of a designer to generate a good solution is shown to be a function of the designer's specific (depth of) knowledge and general (breadth of knowledge). The concept is then transformed into a proposed solution depicted by the set of output variables. It is again noted that the output variables in figure 4 were those of a typical design situation. In general, the output variables are not limited to 'physical configuration' and 'manufacturing process'.

4. **Degree of Constraints** - The constraints affect the transformation of concepts into solutions of the current solutions. As was discussed before, these constraints limit the number of solutions which satisfy all the design requirements. By limiting the possible outcomes, the constraints play a key role in 'shaping' the general form of the solution.

   In such a manner, they resemble dampers in a dynamic system. Just as dampers constrain the motion of a dynamic system, specifications constrain the possible set of solutions. Constraints, in essence, 'damp out' some proposed solutions. And just as dampers tend to promote convergence of dynamic systems to some steady state or equilibrium positions, constraints tend to 'drive' the design process towards a similar set of solutions. Also, dampers tend to decrease the dimension of a chaotic attractor (see Appendix A for discussion on the dimension of an attractor). Since in chapter 1, attractors have been shown to be analagous to a set of proposed solutions, constraints play a similar role to dampers by decreasing the amount of solutions which satisfy the design requirements.

5. **Output Variables** - In the same way that the concept is modeled as a potential energy term, the output variables, such as physical configuration and manufacturing process, may also be modeled as energy terms. For instance, the dynamics analogy of the concept being transformed into a set of output variables that represent a proposed solution, is the potential energy being transformed into useful work. However, constraints limit the extent by which a concept can be applied to solve the design problem. The dynamics analogy to this is that the constraints, acting as dampers, dissipate some energy during the transformation of potential energy into useful work.

   From the skeleton of the mathematical model, there are two types of constraints: the fixed constraints which the total set of requirements impose upon the solution, and varying effective constraints resulting from the coupling between output variables. The fixed constraints do not vary with the progression of the design while the varying constraints depend on the output variables. In other words, the degree of constraint that the manufacturing process imposes upon the physical configuration depends upon what the manufacturing process is. Forging, for instance, imposes different constraints than investment casting. Therefore, fixed constraints can be modeled as *linear* dampers. Similarly, since the varying constraints depend the variable that imposes them, they can be modeled as *nonlinear* dampers.

6. **Degree of Problem Transformation** - After a solution is proposed, it is evaluated against the specifications. Viewing the evaluation process in a simplified manner, two consequences result from an evaluation. Consider a designer's knowledge as some array each element in the array corresponding to a specific knowledge of a different field. Imagine that the value of each element of the array corresponds to a height that represents the designer's depth of knowledge in that area; consequently the designer's distribution of knowledge can be graphically represented as a surface. Through each evaluation, some more appreciation of the problem is gained. This results in an increase in the depth of knowledge to the elements of the array relevant to the overall design problem (the height of the surface in those areas). Also, through a change in perspective, the problem is transformed, consequently drawing upon different areas of the designer's knowledge (different elements of the array). Therefore, perspective acts as a *pointer* to the elements in the array of knowledge. As a designer transforms the problem (views the problem from different perspectives), the specific knowledge he has about each new problem is different.

The evaluation process draws attention to problems associated with a proposed solution. In such a manner, the problem is transformed and a new problem is formulated that if solved, allows the designer to progress towards a solution to the overall problem. Such transformations may lead to increased insight into the problem. Insight can thus be considered as an increase in knowledge through problem transformations.

CONSTRUCTION OF EQUATIONS

With the skeleton which was developed in chapter 3 and the present exposition of the characterstics of the critical elements of the design process, equations describing the fundamental processes of design have been developed. Analogies between the elements of the design process to those of dynamic systems were used to lay the foundation for the mathematical relationships. Equations which were presented at the beginning of this chapter were then proposed to depict the general dependence of each variable on its previous value, other variables, and the process parameters. Now some of the reasoning used in developing these equations will be discussed.

In table 2, the ability of a designer to form good solutions was dependent on the depth of specific knowledge about the particular problem he was solving and on his general knowledge. Considering the concept as analogous to a potential energy term, the general characteristics of table 2 can be depicted by figure 7. The depth of knowlege acts as a magnitude while the breadth acts as an angle from the horizontal plane. The total height (vertical component) represents the potential energy or as in this case, the ability to generate a good solution.

From this representation, the ability to generate a good solution can be defined as:

$$c_{i-1} = k_{i+1} \; sin\phi \tag{1}$$

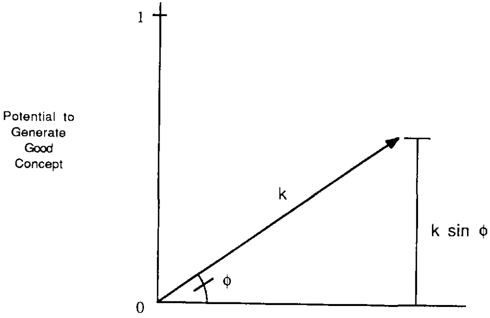

Figure 7. The ability to generate a good solution as the height of a vector whose magnitude is depth of specific knowledge and whose angle (from the horizontal) is breadth of knowledge. The horizontal axis has no significant meaning within the context of this research.

Possible formulations of equations representing the solution (output) variables can then be cast as:

$$u_{i+1} = c_{i+1}(1 - d_1 - b_2 v_i) + (1 - \alpha_i)u_i \tag{2}$$

$$v_{i+1} = c_{i+1}(1 - d_2 - b_1 u_i) \tag{3}$$

where α_i is the degree of problem transformation.

Consider the typical case in which the physical configuration and manufacturing process are the two major concerns of a design solution. In this case, the output variables are u, the goodness of the selected physical configuration solution and v, the goodness of the manufacturing process solution. Since the constraints were shown to be similar to dampers, they dissipate some of the energy being transformed from the potential energy term (c) to the useful work terms (u, v). Note that d_1 and d_2 represent fixed damping (stemming from the total set of design requirements) imposed on the output variables, while b_1 and b_2 are the coefficients of the varying damping acting among the output variables. The first term in the first equation represents the transformation of a concept to a physical configuration. The more rigid the constraints on physical configuration, the higher is d_1. Also, the more the physical configuration depends on the manufacturing process, the higher is b_2. Likewise, if the manufacturing process is highly dependent on the physical configuration, b_1 will be high. If the output variables were completely independent, b_1 and b_2 would be zero representing the case when the output variables u and v do not constrain each other. The same is true of b_2. Now consider the overall effects of coupling between u and v as reflected by the constructed relationships. The better the proposed manufacturing process satisfies the design requirements (higher value of v), the less likely it is to be changed, and the more of a constraint it imposes upon u.

The second term on the RHS of the equation for u reflects the way the new physical configuration depends on the previous configuration. The variable α represents the degree to which the current problem was transformed from the previous one. Thus, if the problem was not transformed to a completely different problem (α is low), it is very likely that the concept did not change much, and that the new configuration is similar to the previous one. The opposite is true when α is high. After a solution is proposed, it is evaluated and again, the problem is transformed. This leads to a proposed equation for α as:

$$\alpha_{i-1} = (1 - u_{i+1})(1 - v_{i+1})\ sin\phi \tag{4}$$

This equation presumes that when the solutions are good (u,v are high), there is little need for major problem transformations. The ideal case occurs when u and v correspond to a perfect solution and there is no longer any need for further problem transformation. Equation 4 also depicts the case when the ideas generated and the solutions proposed are very poor as reflected in low for u and v. In such a situation, the designer is apt to completely change his approach and try to view the problem from new and different perspectives. The $sin\dot{o}$ term represents the fact that a designer with a greater breadth of knowledge is able to transform the problem to a greater extent whereas a designer with a narrow scope of knowledge is less able to transform the problem to one with completely different physical basis.

The equation for specific knowledge is based upon probabilistic reasoning. When there is little transformation in the problem such that the current problem being addressed is similar to the previous one, the designer's specific knowledge of the problem does not change much. Therefore, if the designer's knowledge of a certain problem is deep (k is high), then it should remain high for a related problem. However if the transformation is large to the extent that the newly defined problem is much different, then it is probable that the designer's knowledge is lower for a completely different problem. The opposite is also true. If for one problem the designer's depth of knowledge is lacking, it is probable that if the problem is transformed to a completely different one, his knowledge will be higher. This rationale is depicted in table 3.

Table 3. The variation of specific knowledge, k, as a function of its previous value and the degree of problem transformation from i to $i+1$, α

	Low k_i	**High** k_i
Low α_i	k_{i+1} is low	k_{i+1} is high
High α_i	k_{i+1} is high	k_{i-1} is low

The following equation was the simplest equation to portray the general characteristics in table 3:

$$k_{i+1} = \lambda(k_i - \alpha_i)^2 \tag{5}$$

where λ is a 'gain' parameter representing the designer's ability to learn. If k_i is high and α_i is high, then the square of the difference of k_i and α_i is low and therefore k_{i+1} is low. This equation models the general trend of the other three cases in table 3. The squared difference is utilized to produce only positive quantities representing the difference between the variables.

FURTHER DEVELOPMENT OF THE MODEL

Through equations (1-5), the model crudely portrays some of the fundamental relationships between the critical design variables. However, it has a few shortcomings and therefore further development of the model was needed. First and foremost, attempting to describe the variation of specific knowledge as a function of the degree of problem transformation was difficult. The equation for k in the model discussed above is based purely on the probability of the designer's knowledge increasing or decreasing with respect to the specific knowledge of the previous problem. If a designer's general knowledge is considered a large array with each position in the array representing a different field of knowledge, then the value at each position depicting the amount of specific knowledge corresponing to that field. It may be convenient to picture this array as a plane with the specific knowledge values forming a contour map over the plane. Corresponding to a specific knowledge, α then becomes the pointer to the position of the array corresponding to a specific field. It is then obvious that such a map is very different for each designer and that it may be difficult to find a general characteristic common to how the specific knowledge of any designer varies with the degree of problem transformation.

Also, the manufacturing process variable, v, was treated differently than u by not being a variable of itself. However, when considered carefully, in many cases the manufacturing process itself is designed. As a matter of fact, an entire design problem may focus on the design of a manufacturing process to fabricate some structure. This may often be the case, for example, when designing with composite materials (17). Therefore, the dynamics of the output variable, v, should be similar to u.

The approach taken to address some of the problems with the first model (equations 1-5) centered around two main actions. Firstly, the equation for v is now written such that it is similar to u:

$$v_{i+1} = c_{i+1}(1 - d_2 - b_1 u_i) - (1 - \alpha_i)v_i$$

However, it soon becomes evident that it may be simpler and more meaningful to just combine the two output variables and replace them with one variable which represents the goodness of the applied solution in general. This new variable, called x, will now encompass the entire applied solution (configuration, manufacturing process, and any other relevant aspect of the solutions).

Secondly, consider a better representation of how specific knowledge varies with problem transformation which overcomes some of the awkwardness of equation 5, namely, consider:

$$k_{i+1} = k_i + k_i(0.5 - \alpha_i)(1 - k_i)$$

If $\alpha = 0.5$, it signifies no change in specific knowledge. As α approaches 1, the second term in the equation is negative and k decreases. The terms k and $(1 - k)$ in the second term impose a bound on the range of k from 0 to 1. Since the first term of the RHS is k_i, the equation essentially describes $(k_{i+1} - k_i \;=\; f(k_i,\ \alpha_i))$ the *change* in specific knowledge (Δk) as a function of problem transformation. Thus α takes on a slightly different meaning. Now it represents a pointer to the change of specific knowledge. The model now evolves to:

$$k_{i+1} = k_i - k_i\alpha_i(1 - k_i) \tag{6}$$

$$c_{i+1} = k_{i+1}\ sin\phi \tag{4}$$

$$x_{i+1} = c_{i+1}(1 - d - bx_i) + (1 - \alpha_i)x_i \tag{7}$$

$$\alpha_{i+1} = (0.5 - x_{i+1})\ sin\phi \tag{8}$$

Although this model offers several advantages over the first model of equations 1-5, *some weaknesses still remain*. The new $k(k_i, \alpha_i)$ equation is more meaningful since it depicts the gain and loss of specific knowledge, rather than the absolute specific knowledge. However, the equation also introduces two new problems: Firstly, the equation for α may not suit its new role in determining the change in specific knowledge. More importantly however, the equation for k imposes undesired dynamics upon the model. As k increases, c and x increase further, thus increasing k. The opposite is also true. Therefore, k will either approach 0 or it will approach 1, depending on the initial conditions.

Until this point, the shortcomings of most of the models which were studied centered around the definition of α and k and the relationship between them. This issue became the focus of further development.

The degree of problem transformation (α) may not be the most meaningful variable to convey the dynamics of the evaluation process. Also, it is very difficult to describe how the specific knowledge of the designer varies with degree of transformation. When considering the variables, problem transformation, α, and specific knowledge, k, together, their roles were to portray the gain or loss of knowledge as a function of the proposed solutions and the design process parameters. In other words, they were to characterize the amount of *insight* the designer has into the current design problem. With this in mind, a variable θ defined as a measure of insight into the problem, is best introduced to replace α. Basically, insight in the context of design is the knowledge gained from the simplication of the problem by viewing it from a different perspective (12).

Now that θ is defined as a measure of insight, the dependence of k on θ is much simpler and more identifiable. In fact, there really is no need for a separate variable k describing specific knowledge. One more variable and corresponding equation can be eliminated from the model. Now the problem of defining how θ is a function of the other variables and parameters must be addressed. There are several straight-forward relationships between θ, c, and x that can be readily identified. For example, although it is a rare occurence, sometimes a designer may stumble upon a good configuration although his concept is poor. Upon evaluation, however, an effective designer will gain insight by identifying the reasons why the configuration is good. In other words, the good designer will extract the underlying concept behind the good configuration. This new insight may allow him to generate a better configuration from the identified concept or it may kick him into a whole new family of concepts.

In fact, a designer may gain more from evaluating a good configuration originating from a poor concept than the converse situation, a poor configuration from a good concept. The reasoning behind this argument is that since the generation of a configuration is our final objective, it is the ultimate form of insight. Often a designer may have a good concept which he then finds difficult to apply towards a solution. For instance, a designer may have a good idea to solve some given problem. However, he may find that new problems are posed by the constraints which were not apparent to him before, since he is now approaching the solution in a different manner. In other words, the concept is a *basis* for a solution; a good concept provides high potential for a good configuration. However, a good configuration is *a solution* which satisfies the constraints and solves the design problem, thus providing a clearer understanding of the principles which are involved in the solution of the problem. Therefore a good configuration serves as a strong source of insight.

A final observation is made before arriving at the equation related to insight. Although the generation of a good concept and a good configuration are an indication of the designer's insight into the problem, the designer will not gain much more insight once he has a solution for the problem. This case represents high insight but low gain of insight. In the opposite case, a poor concept and configuration indicate low insight and also lead to a low gain of insight. From the above characterisitics, the following simple equation for θ, the *gain* of insight, can be written:

$$\theta_{i+1} = x_{i+1}(1 - c_{i+1})$$

The new definition and equation for θ also depict another characterisitic of the design process which the previous models could not portray: insight into the problem is always gained. Even if the knowledge of the designer may decrease when the problem is transformed, the overall insight he has into the problem is always increasing.

The new equation which represents the ability to generate a good solution becomes:

$$c_{i+1} = \lambda(1 - d - x_i)(c_i + \theta_i \, sin\varphi)$$

where λ and φ are parameters which relate to the designer's ability to learn from evaluation and apply insight, and the designer's general breadth of knowledge, respectively. φ ranges from 0 to $\frac{\pi}{2}$ while λ ranges from 0 to approximately 3, depending on the other parameter values. This range is determined by the extreme values of λ that still bound the model's dynamics. The constant d remains to measure the degree of constraints imposed by the specifications and ranges from 0 to 1 where 0.05, for instance, may mean low constraints and 0.4 means rigid constraints.

In the first discarded model, c was defined as $k \, sin\phi$ such that $\Delta c = \Delta k \, sin\phi$. Since θ now represents a change in knowledge, Δ, the second term in paranthesis in the new equation for c is similar to $c + \Delta c$. In other words, the new concept is a function of the old concept and the new insight gained. The (1-d-x) term is needed in the equation to characterize that a good configuration may also hinder the designer from generating new concepts since it promotes fixation.

Next, consider the modified equation for the applied solution:

$$x_{i-1} = c_{i+1}(1 - d(1 - c_{i+1}) - \theta_i)$$

This equation is similar to those of previous models. Here, the insight term, θ, directly affects the application of a solution as it does the concept. The (1-c) term multiplied to d adds a new feature. As the problem is transformed, the effect of the constraints may vary. A large gain in insight indicates that the problem has been transformed such that the effects of the constraints is minimized. In the shaft brake system example discussed in chapter 4, when the problem was defined as a device to slow the motion of the shaft, wear immediately became a problem since the specifications required long life. The problem of wear severely limits the total set of possible solutions. It was only when the problem was transformed to its most basic or critical form, removal of energy from the shaft, that a large host of configurations were made possible. In other words, although the life constraint was fixed, its effects on the design process vary with the definition of the problem and the concept. If the problem was boiled down to the solution of wear, the life specification heavily constrained the possible solutions. However, when the problem was transformed to the removal of energy from the shaft, the problem of wear was avoided altogether which lessened the constraining effect of the design requirements. This made many more solutions possible.

After redefining θ as the gain of insight, the problems of attempting to depict absolute specific knowledge were eliminated. Also, since θ is always positive, the insight into the poblem is always increasing which had been difficult to simulate with the variable k and the older definition of θ (degree of problem transformation).

Again the final model becomes:

$$c_{i+1} = \lambda(1 - d - x_i)(c_i + \theta_i \, sin\phi) \tag{9}$$

$$x_{i+1} = c_{i+1}(1 - d(1 - c_{i+1}) - \theta_i) \tag{10}$$

$$\theta_{i+1} = x_{i+1}(1 - c_{i+1}) \tag{11}$$

Note that the initial values of the variables represent the beginning of the design process. For example, θ, the gain of insight from the evaluation of a solution, is set to 0 at the start of the process. The value of c will represent the goodness of the initial concept, while the initial value of x is calculated.

Besides, those already discussed, there are several other advantages to this model over the others. Most important is its ability to capture the fundamental characterisitics of the design process in a simple set of three equations.

Consider the resemblence between the present three variable model and March's scheme of the design process (see figure 8). March attempts to depict design as a continuous sequence between three rational processes. The equation of x from c denotes the creation and embodiment of a solution from a concept. This is essentially what March identifies to be production, or as he sometimes refers to it, abduction. Once a proposed solution is generated, it is analyzed. In March's scheme, this is a deductive process since we use our factual knowledge of physical princples to analyze a proposed solution and its functional characteristics. Finally, through the results of evaluation of a proposed solution, we generalize and draw inferences to increase our knowledge of the overall problem. This is represented in the model by the use of gained insight, θ, to formulate new concepts.

By manipulating the equations of the model, one equation may be eliminated. From the third equation, an expression for θ can be obtained and substituted into the second and third equations which will ultimately yield a two equation model:

1. $c_{i-1} = f_c(c_i, x_i, \lambda, \phi)$

2. $x_{i+1} = f_x(x_i, c_{i-1}, c_i, d)$

The two equations can be considered a mathematical model depicting the processes occuring in Jansson's two space scheme (see figure 9). The first equation represents 'generalization', the movement from configuration space to concepts space. The second equation portrays 'realization', the movement from concept space to configuration space.

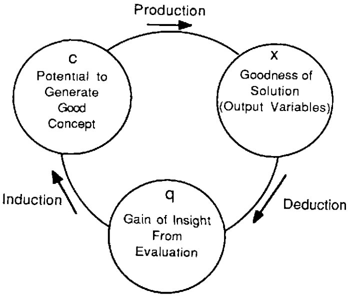

Figure 8. Three variable model and how it fits within March's depiction

In the next chapter, a brief mention of the dynamics of the model will be included along with an interpretation as related to design.

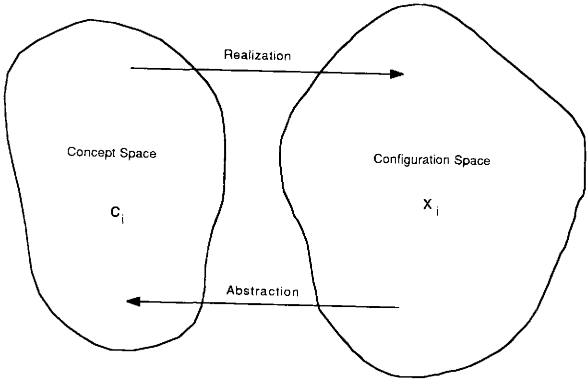

Figure 9. Reduced two variable model and how it fits within Jansson's scheme

CHAPTER V

THE DYNAMICS OF THE MODEL

The behavior of a nonlinear dynamic system may vary drastically depending on the values of its parameters and the intitial conditions. The model to be studied is a discrete mapping as opposed to a set of continuous differential equations. For a given set of parameter values, the dynamic behavior is displayed in a two dimensional plane of paired state variables.

The effects of the three parameters, (λ, ϕ, and d) will be examined to test the ability of the model to simulate general characteristics of the design process. The plots represent the maps of c, the ability of the designer to generate good concepts, and x, the goodness of the solution. Although, the nature of the attractors exhibited by the model will be discussed later, a general guide to the interpretation of the maps are as follows:

When the model settles onto a fixed point or set of fixed points where the magnitudes of c or x are relatively low, this represents a state of design fixation.

When the model settles onto a fixed point or set of fixed points where c and x are relatively high, this represents the convergence onto a good solution.

A chaotic regime of the model represents a dynamic design process in which each idea leads to another new idea.

Note that appendix A which contains a discussion of the basic underlying mechanisms behind chaos and terminology relevant to this research has been added for the convenience of the reader. Consider first the plotted maps of the final model for three different cases of λ (low, medium, and a high) for the same values of ϕ and d ($\phi = 50$, $d = 0.15$) in figures 10– 12.

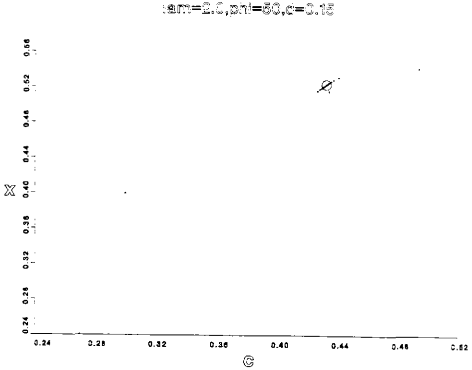

Figure 10. Map of case: λ=2.0,ϕ=50,d=0.15

$\underline{\lambda = 2.0}$

For this value of λ, the model yields a fixed point (fixed points will be circled to differentiate them from transient points). Since the values of c and x are low (not close to 1), this case represents that the process has converged onto a poor solution as a result of an early fixation. This means that the design process has converged onto one set of ideas or solutions which do not satisfy the design all of the requirements. To jump out of such a state, some action must be taken, such as viewing the problem from a different perpective and change the approach entirely or seeking more information that may increase insight into the problem. In terms of the model, the process will remain at that point until an external input perturbs the system or the parameter

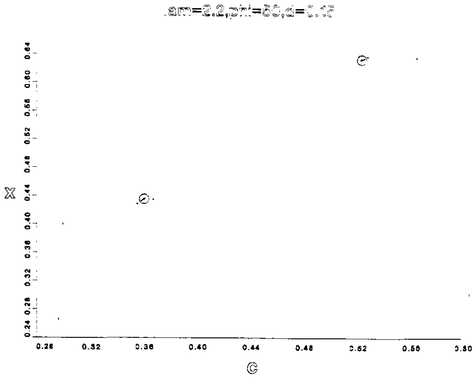

Figure 11. Map of case: λ=2.2,ϕ=50,d=0.15

values are changed. In either the model or an actual design situation, some active change is required to kick the process out of a fixated state.

$\lambda = 2.2$

For the same ϕ and d, when the value of λ is increased to 2.2, a bifurcation occurs and the system converges onto a period-2 regime. This may correspond to a designer with a greater ability to learn from the process of evaluation and problem transformation. Under the general conditions depicted by the other parameters, such a designer may arrive at two proposed solutions. The right - top point represents a better solution than the one arrived at by the designer of case λ = 2.0.

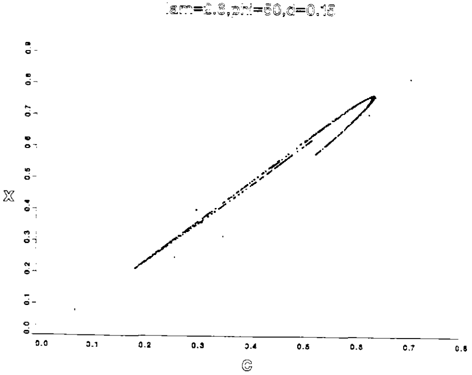

Figure 12. Map of case: λ=2.6,ϕ=50,d=0.15

$\underline{\lambda = 2.6}$

A designer whose learning ability corresponds to a $\lambda = 2.6$ has a greater ability to visit more and better solutions. The attractor depicted in this case is chaotic in which each point yields another new point. At this stage, the exact meaning of the shape of the attractor is undetermined; however, it is apparent that this designer is able to generate more concepts and not become fixated on any one solution but to learn from the evaluation process and propose other solutions.

Now consider the effect of parameter ϕ for $\lambda = 2.8$, $d = 0.25$ (figures 13–15).

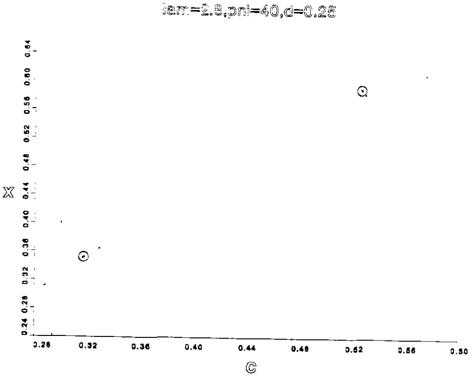

Figure 13. Map of case: λ=2.8,ϕ=40,d=0.25

$\underline{\phi = 40}$

Under the general conditions depicted by $\lambda = 2.8$, $d = 0.25$, the case in which the designer's breadth of knowledge corresponds to $\phi = 40$ results in two solutions. Again, as was the case with λ, the plots will demonstrate the effects of different values of ϕ. In this study, absolute values of ϕ have not yet been correlated with a specific level of breadth of knowledge.

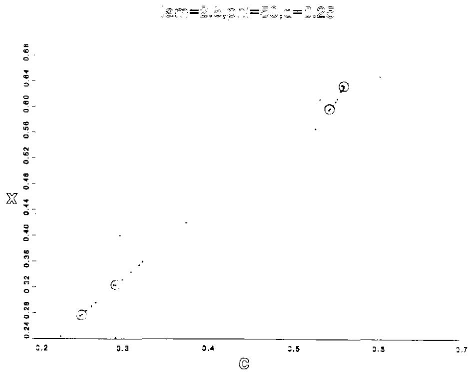

Figure 14. Map of case: λ=2.8,$\wp$=50,d=0.25

$\underline{\wp = 50}$

As the diversity of the designer's knowledge increases, greater number of solutions are generated. Also, since such a designer's breadth of knowledge allows him to attack the problem from many different perspectives, he is able to generate a wider range of solutions, some of which are better than those generated by the $\wp = 40$ designer.

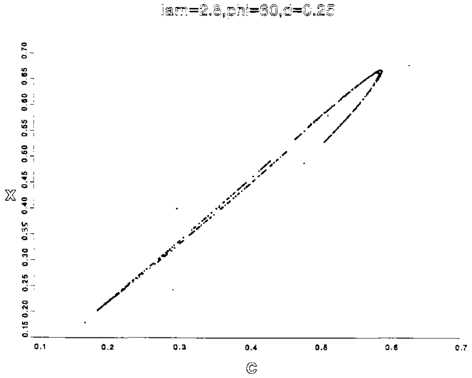

Figure 15. Map of case: λ=2.8,ϕ=60,d=0.25

$\phi = 60$

A greater ϕ further increases the number and the range of goodness of the solutions. Notice it is not the absolute goodness but rather the *range* of the solutions that increases with the breadth of knowledge of the designer. This implies that such a designer has the ability of visiting a greater variety of solutions in general. This seems reasonable since the designer whose knowledge is more diverse is able to identify approaches to the solution of the problem. It is thus expected that he can generate a wider spectrum of solutions.

Finally, the effects of parameter d will be examined (figures 16–18).

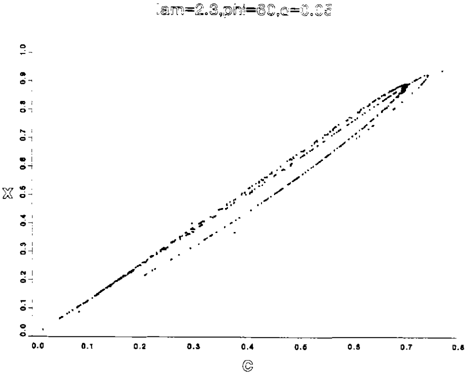

Figure 16. Map of case: λ=2.3,ϕ=60,d=0.05

The first situation poses a problem which is loosely constrained with a designer described by $\lambda = 2.3$, $\phi = 60$. Because of the low value of d, it is easy for this designer to generate many alternatives for such a problem. The process is chaotic representing that the design is in a fruitful state of divergence at the conceptual level. Notice that the process now visits more points of high c and x which depict proposed solutions which satisfy the design requirements to a greater extent.

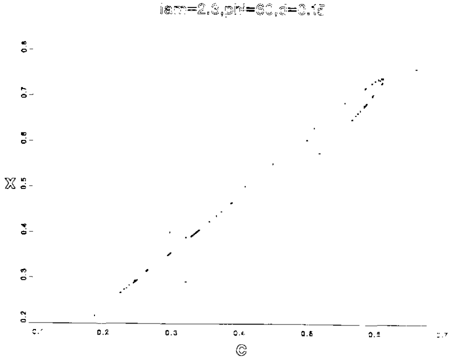

Figure 17. Map of case: λ=2.3,ϕ=60,d=0.15

<u>$d = 0.15$</u>

Consider now the case of a more constrained problem with the same designer. In this case, he is able to generate fewer solutions which satisfy the constraints posed by this new problem. Although the design process may also be chaotic in this situation the *dimension* of the attractor is less than in the previous case for d=0.05. If the dimension of the attractor can be viewed as a measure of creative productivity, a sparser attractor signifies a less fruitful process. The analogy between constraints and dampers is made more apparent through this case. Increasing the damping decreases the dimension of the attractor.

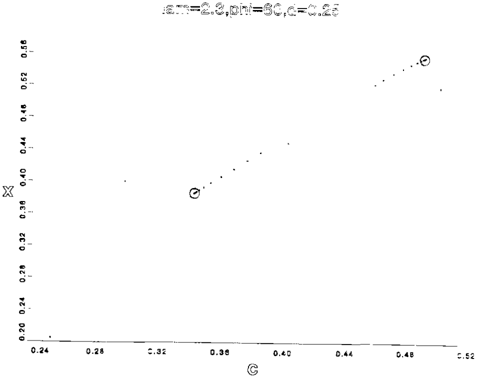

Figure 18. Map of case: λ=2.3,ϕ=60,d=0.25

$\underline{d = 0.25}$

The constraints in this situation pose a more difficult problem for the designer. The problem is constrained in such a way that the designer depicted by $\lambda = 2.3$, $\phi = 60$, is not as capable of productive lateral thinking.

Again note that these plots only represent the case of 'an isolated' designer with no available resources. In the dynamic sense, these plots only reflect the steady state cases and do not yet show the transient effects of new information and other transients caused by continuously changing parameters.

DISCUSSION AND SUMMARY OF RESULTS

The general results generated using the present mathematical model of the design process confirm those fundamental relationships which were identified in the qualitative model skeleton. Most obvious is the dampening nature of the constraints (parameter d) on the dynamics of the design process. A design problem modeled with a very low parameter d (low degree of constraints) may lead to good solutions (high c and x) without requiring high λ and o parameter values. In other words, very simple and barely constrained problems may not require the ability to scan a broad range of solutions as much as highly constrained problems. The converse, as shown by the model, is also true. It takes designers with broader knowledge and a greater ability to fruitfully learn from problem transformations to solve problems which are highly constrained.

The types of behavior expected to be exhibited by the model may be categorized by the following types of attractors.

1. Period-n - representing a periodic regime. The n^{th} point maps back to the first.
2. Closed Orbit - representing a quasiperiodic regime. No points map back to any previous point.
3. Strange - representing a chaotic regime.

Each iteration of the model represents a cycle through the skeletal (qualitative) model discussed in chapter 5. Therefore, each iteration of the model represents the identification and definition of a problem, the formulation of an idea or conceptual solution, and the application and embodiment of the concept to a configuration. Within this context, when the model converges to a fixed point, it translates into one of two different situations: either a *solution* which satisfies the requirements, or a point of *fixation*. Before proceeding further, consider the difference between a solution and a point of fixation. In both cases, the designer will be converging onto a single state or set of states. Perhaps the only difference between the two cases is that the convergence onto a good solution is desirable. On the other hand, the convergence due to fixation is undesirable since the state of the design at that point does not represent a solution which satisfies the requirements. Hence, both situations are naturally depicted by a fixed point; however, the nature of the fixed point differentiates the two cases. In fixation, the goodness of the concept and configuration, represented by the values of c and x, is not sufficient to form an 'actual solution' to the problem. At this point in the research, the relative values of c and x which represent a solution that satisfies all the requirements has not yet been determined.

To get out of a design fixation, some forced external action, usually in the form of a decision to change something in the approach to the solution must be taken. This is usually done by viewing and attacking the problem from a different angle and re-examining how the problem is defined. Perhaps the problem is defined in such a way that there are too many constraints on the solution. Perhaps a pre-conceived notion of a solution must be discarded and the solution re-approached 'from scratch'. The best way to jump out of a fixated state is usually to transform the problem and study it from a different angle. The situation is the same when the model depicts a fixated state; some external impulse must be input to the process or else it will remain in a fixated state.

The ways by which this can be done in the model correspond closely to an actual design situation. One possibility is to change the parameters representing some change in the design team, perhaps the addition of a new member. Another way is to impose a shift in θ corresponding to some gain in knowledge from available resources (information) or from a transformation in the problem. This represents the gain of insight through consultation with another person or through relevant research.

Consider the extreme opposite case. When the model, for certain values of the parameters, yields a chaotic attractor, it represents a very dynamic design process. On a map, a chaotic attractor will appear as a large set of scattered points which do not form any Euclidean geometrical shape (point, line, or circle) yet are not randomly distributed. In a chaotic attractor, each point or state yields a new and different state. In other words, each point may correspond to a different solution or a different state of one solution. In the dynamics sense, such a case represents a divergent process. In design, this translates to the exploration of multiple avenues for a solution. For instance, a productive brainstorming session is a good example of a chaotic or divergent process since one idea may lead to the generation of another idea.

Now, a speculation is made of how periodic regimes are analogous to design. In the strictest sense, a regime of periodicity n depicts a serial evolution from point 1 to 2 to...n back to 1 and so on. However it is important to note that the entire set of points constitute a single attractor since any nearby trajectory attracted to one of the fixed points will not stay at that point; rather, it will cylce among the n points. Otherwise, it may be wrongly translated into one proposed solution (attractor) leading to another proposed solution (attractor) and so on. Instead, the entire set of states coexist as a single attractor. In other words, they are not to be considered as separate autonomous states but as a collection of points which represent a single state.

When a periodic attractor is misinterpreted as a serial sequence of autonomous fixed points, it becomes difficult to match it to a design situation. However, when the n fixed points in a period-n attractor are considered as an entity, they may represent the situation when n simultaneous solutions have been proposed. In this sense, the set of solutions exist in a single design state. Another interesting observation can be made. A dynamic system with any one set of parameter values may have multiple, say m, attractors. The one which the system will approach is solely a function of the initial conditions. This situation is different from a single period-n attractor since here, each point or set of points constituting an attractor is autonomous. If for a given set of initial conditions, one of the m attractors are approached, it translates to the design situation in which the designer is approaching a proposed solution and may be unaware of the other $(m-1)$ solutions. This is in contrast with a period-n attractor in which the designer has generated a set of n alternative solutions. To clarify this further, for a given set of parameter values, the model may have for instance, $m = 3$ attractors, two fixed points and one period-4 attractor. For a given set of initial conditions, the system (designer) may approach any one of the attractors. In other words, he may approach either of the fixed points (single proposed solutions) or the period-4 attractor (a state of four coexisting alternatives). When considered in this manner, a period-n attractor is one set of simultaneously existing solutions. Therefore, a high periodicity attractor naturally represents a *parallel* search for solutions.

An obvious general conclusion may be drawn from the behavior of the model by considering the case of a fixated state depicted by a fixed point of low c and x on the map. This could happen quite frequently in the case of a highly constrained problem. However, the desired destination for the designer is a point of high c and x. Consider now the routes which may lead from the fixated point attractor to ta point attractor representing a good solution:

1. Some change in the variables, an externally imposed shift in θ denoting gain of information through insight for example, may kick the system out of the fixated point attractor and into the *basin of attraction* of the solution point attractor. Note that a basin of attraction is the set of all initial conditions that will ultimately lead a dynamic system to a given attractor (see appendix A).

2. A change in parameters which pushes the system into a chaotic regime such that a number of different points are visited. When it reaches a good solution (a point of high c and x), another change in parameters can push the system back to convergence, this time on the solution point attractor.

3. A change of parameters might cause an unstable periodic regime to to collide with a chaotic attractor resulting in a sudden expansion of the chaotic attractor and possibly encompassing a set of good solutions. In modern dynamic theory, this is called an interior crisis.

The last two paths may shed some light on the global role chaos plays in the design process. Chaos may be viewed as a *route* for jumping from a fixated state to a good solution. In other words, through the dynamics of chaotic behavior, the process may visit points which represent good design solutions.

CHAPTER VI

SUMMARY AND FUTURE RESEARCH

Design is clearly an extremely complex process. As was discussed in the introduction, there is ample evidence to suggest that strong similarities exist between the design process and chaotic dynamic systems. And since the simplest of chaotic systems can exhibit complex, unpredictable behavior, chaos may provide a means to mathematically model the general dynamics of the design process with simple equations.

After identifying the critical factors that form the underlying structure of design, some simple equations were written describing the processes between them. The approach taken viewed the design process from a global perspective. Instead of attempting to model all of the specific details, only the most fundamental processes common to all levels of design were considered.

If the model is sound in its description of the fundamental processes, and is capable of displaying chaos, the complex behavior of the model may intrinsically describe the complex behavior of the design process. From studying such a model, not only will we gain much insight into the dynamics of the design process, but we may be able to draw some general conclusions about the most effective and efficient ways to approach the solution to a design problem. From the model, for instance, we may learn, for a problem with a certain degree of constraint, when to scan for general concepts and when to focus and develop a concept, when we may not have ample resources, and when the process is no longer sensitive to external information.

Also, since there is order much order in the unpredictable behavior of chaotic systems, we may be able to identify certain order within the complexity of design which may further enhance our understanding of the process. For chaotic dynamic systems, this order is reflected, for example, in the specific shape and fractal dimension of the attractor; the specific routes to chaos through various types of bifurcations and various well recognized phenomena of sudden changes such as crises and intermittencies.

FURTHER RESEARCH

1. The next logical step is to develop a basis and method to quantify the values of the parameters and initial conditions of the variables; a designer's breadth of knowledge, for instance, must be quantified and represented as a single number from 0 to $\frac{\pi}{2}$. Once a technique is devised to assess the values of the parameters and variables, the model may be used to evaluate actual design activities and provide a means by which to guide the designer's course of action.

2. Now that the initial model has been generated, some tests may be conducted to generate feedback on the validity of the model. Some carefully planned experiments may be designed to verify the ability of the equations to model actual design activities. The feedback may then be used to modify the equations and increase the accuracy of the model. The details of the intricate dynamics exhibited by the model are yet to be analyzed. For instance, the causes for the exact shape, size, and dimension of the attractors must be determined

3. Many questions remain to be answered. For example, what is the significance of quasiperiodic behavior, yet another possible attractor? Quasi-periodic behavior constitutes a relevant aspect of multi-dimensional maps which result in rich, complex behavior. How is it related to the design process? What is the significance of the exact size and shape of an attractor? Through more study of the model and of conducted experiments, the relevence of these issues may be determined.

4. Although self-similarity is one of the properties that led to the investigation of chaotic equations to model the design process, its role in the design process has not yet been determined.

5. A crises is a sudden change in the size and dimension of a chaotic attractor resulting from a parameter change. From the perspective of design, a crisis signifies when new solutions are generated or existing ones are discarded. Further study of crises as related to design may reveal other advantageous features of the model in describing characterisitics of the design process.

6. The whole process of developing the qualitative and mathematical model in this study was in itself a design process. Therefore, it may prove interesting and insightful to view the construction of the model in retrospect and determine its evolution within the context of the mathematical model.

Although this research started with exploring the use of chaos to mathematically model the design process, the model reveals that chaos may be a desirable mechanism to achieving good design solutions. The role of chaos may then shift from the descriptive role of depicting the complex activities of the design process with simple equations to the prescriptive role of providing a route to good design solutions. Once the techniques are developed to assign parameter values to actual design situations, the model may serve as a 'monitor' to the design process. With such a powerful tool, the ultimate goal of future research is to determine how to achieve the change in parameters which will lead to chaos in an actual design situations.

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APPENDIX A

BRIEF OVERVIEW OF NONLINEAR DYNAMICS AND CHAOS

BEHAVIOR OF NONLINEAR SYSTEMS

Before overviewing relevant nonlinear dynamics theory, some basic terminology must first be introduced. When studying a dynamic process, we are usually interested in the time evolution of the system. Often, a dynamic system is modeled as an n^{th} order differential equation which can be reduced to a set of n first order differential equations. The set of first order differential equations is termed a *flow*. One common technique to visualize a flow is by plotting a state variable against its time derivative. This is known as a *phase space diagram* (8) . For any initial conditions, the curve describing the time evolution of a system in phase space is called a *phase trajectory*. A set of phase trajectories is a *phase portrait* (see figure 19.)

Viewing a phase portrait for flows of three dimensions or higher may become confusing. Henri Poincare (19) devised a method whereby an arbitrary plane section is constructed to intersect a three dimensional flow (or a 3 dimensional subspace of a flow). Each time the flow intersects the plane, it will appear as a point. Each point will then 'map' onto another point where the flow next intersects the plane. The set of points on such a plane is termed a Poincare section (see figure 21). A 3 dimensional flow can thus be represented by a two dimensional iterative mapping.

The *fixed points* of a system executing small motions about a static equilibrium position are the equilibrium positions. In linear dynamic systems, the fixed points can be categorized as stable fixed points, unstable fixed points, saddle points, or centers and foci. The nature of the fixed points can be determined by the *characterisitic exponents* of the system. How the characteristic exponents are calculated will not be

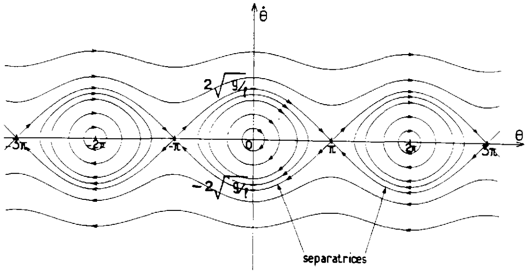

Figure 19. Phase Portrait of a simple pendulum where θ is the angle from the vertical and $\dot{\theta}$ is its time derivative. From (8)

Table 4. Nature of fixed points of 2D linear systems.

Real(μ)	**Imaginary(μ)**	**Nature of Fixed Point**
-	none	stable fixed point
+	none	unstable fixed point
+ and -	none	saddle point
-	pair	stable focus
+	pair	unstable focus
0	pair	center

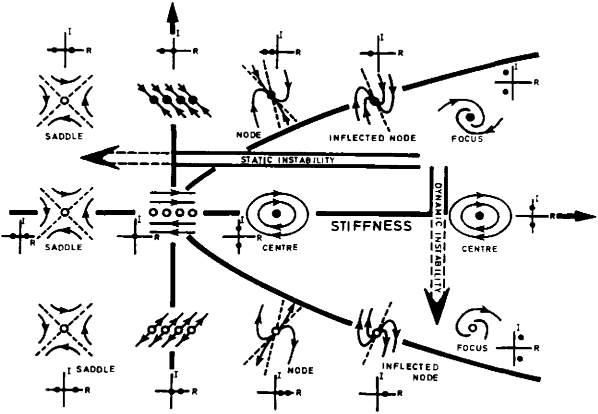

Figure 20. 2D Linear system fixed points. From (18)

discussed here. Suffice it to say that an n-dimensional system has n characterisitic exponents for each fixed point. Table 4 summarizes the nature of fixed points of a two dimensional system as related to the characterisitic exponent (μ). Figure 20 depicts the different types of fixed points.

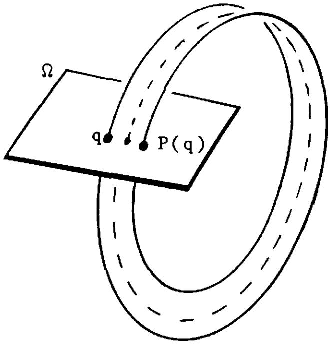

Figure 21. Points on a Poincare section. From (20)

An unstable fixed point repels nearby trajectories and can thus be termed a *repellor*. A stable fixed point, on the other hand, 'attracts' nearby trajectories. The existence of an *attractor* then suggests a dissipative system since energy is being removed from the system until it rests in low energy equilibrium position. For each attractor, there is a corresponding set of initial conditions that will lead the system to the attractor. This set of initial conditions is termed a *basin of attraction*.

A conservative or Hamiltonian system is time independent and energy invariant. This can be readily seen in phase space. Consider the trajectories emanating from a set of initial conditions forming a square in phase space. If these trajectories are traced forward in time for a Hamiltonian system, the corresponding points may form a shape other than a square but the area will be the same as the original square. In other words, areas in phase space are conserved for Hamiltonian systems. Similarly.

phase space areas decrease for dissipative systems. Therefore, in the vicinity of a point attractor, the trajectories converge. On the other hand, they diverge in the vicinity of a point repellor.

Nonlinear systems can exhibit much more exotic behavior. One characterisitic of nonlinear systems which contributes to their more complex dynamics is their ability to *bifurcate* and yield multiple solutions (18). In the most general sense, a bifurcation is a qualitative change in behavior by varying a system parameter or a set of paramters. For instance, with the variation of a parameter, a stable fixed point may become unstable and give way to two new stable fixed points. An example will be discussed later which illustrates this type of bifurcation. A stable fixed point may also go unstable and yield a stable *limit cycle*; this is known as a Hopf bifurcation. A limit cycle is a closed orbit attractor in phase space as opposed to a single point attractor. In figure 22, the limit cycle of a Van der Pol oscillator governed by the equation $\ddot{\theta} - (\gamma - \theta^2)\dot{\theta} + \theta = 0$ is shown.

A Poincare section which is constructed such that each point maps onto another point at a period of $2\pi/\omega$, where ω is the forcing frequency of the flow, is called a first return map (figure 21). The first return map of a limit cycle is a point since the orbit intersects the plane at the same point every revolution. By changing a parameter, the limit cycle becomes unstable and a bifurcation to another stable orbit occurs. If after a bifurcation, the single limit cycle gives way to two closed orbits which now intersect the Poincare section at two points, a period doubling bifurcation has occurred. The bifurcation is easy to see on the first return map. The stable fixed point representing a limit cycle becomes unstable and yields two new fixed points (see figure 23) A *cascade* of period doubling bifurcations can continue into regimes of higher periodicity.

As long as the ratio of the flow frequency to the forcing frequency is rational, the flow is periodic. On a first return map, this is indicated by a finite set of points which eventually revisit each other. For example, when the ratio between the two

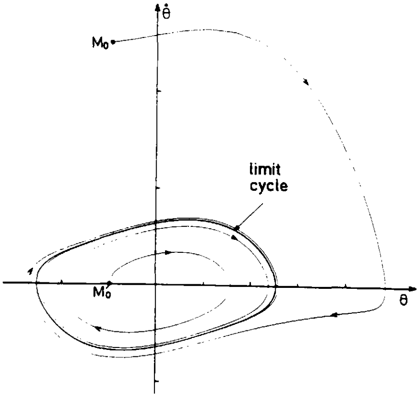

Figure 22. A limit cycle in a Van der Pol oscillator. From (8)

frequencies is 4, then the fourth point will map back to the first point. Consider when the ratio of the flow frequency to the forcing frequency is an irrational number. In such a case, never will any point map back to a previous point. The result is what appears to be a closed curve or a *quasiperiodic* attractor on the map which signifies that the phase space flow is in the shape of a T^2 torus depicted in figure 24 and results from a Neimark, or secondary Hopf bifurcation.

CHAOS

In nonlinear systems, the types of attractors include but are not limited to fixed points. Other attractors discussed thus far are limit cycles (periodic attractors) and T^n tori (quasiperiodic attractors). One common characterisitic of attractors of nonlinear systems is *boundedness*. Unlike linear systems in which an unstable fixed

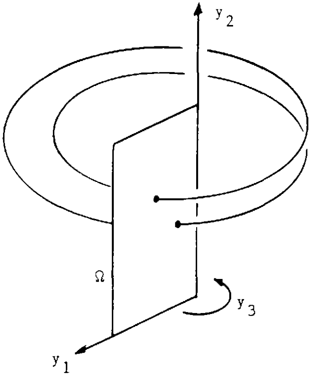

Figure 23. A single period regime bifurcating to a period two regime. From (20)

point can repel trajectories to infinity, nonlinear systems impose a boundedness on the flow. So if we consider a 3 dimensional system, trajectories in the attractors discussed thus far are always converging in at least one direction. In this discussion, 'direction' does not conote a global axis but rather some local direction varying with the flow. Trajectories approaching a fixed point converge from all three directions, while in the limit cycle they locally converge from all but one direction, and so on. This can be generalized for an n dimensional system. One measure of the average rate of convergence of nearby orbits in phase space along each of these 'local directions' is the *Lyapunov exponent* (21).

There are n lyapunov exponents (λ) for an n dimensional system. A negative λ indicates the exponential rate of convergence. If a flow contains at least one positive λ, then trajectories are locally diverging. However, since nonlinearity enforces

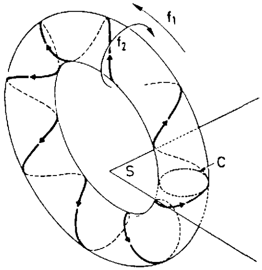

Figure 24. Motion on a torus - quasiperiodic regime. From (8)

boundedness, the trajectories must also be converging along another direction. This bounded divergence is what is refered to today as *chaos*. In figure 25, the trajectories of the Rossler attractor demonstrate the continuous divergence and convergence. This

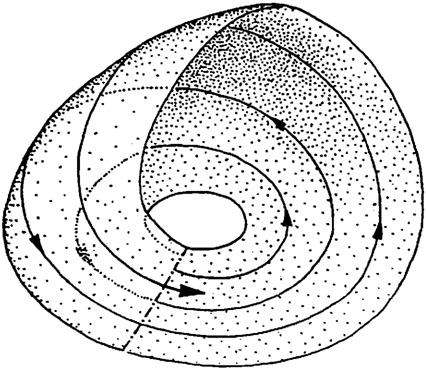

Figure 25. Continued divergence within a bounded space - The Rossler Attractor. From (22)

motion is referred to as *stretching and folding.*

A chaotic or *strange attractor* on the whole is stable and attracts nearby outside trajectories. However, as is easily seen from a map, the attractor is involves an infinite number of unstable fixed points and also saddle points, attracting trajectories from one direction and then repelling them in another. It is this stretching and folding which yields the properties of a chaotic system. Stretching refers to the divergence of trajectories which implies sensitivity to initial conditions. A very small perturbation becomes magnified out of proportion. The folding then produces a mixing effect in which trajectories are brought back together again to keep them within a finite space. The combination of stretching and folding yields sensitivity to initial conditions within a bounded space. This leads to unpredictable behavior. Consider a trajectory which is a small, finite, distance apart from another trajectory. It will continually diverge within the bounded space until its behavior no longer resembles the first trajectory. Therefore any imprecision in the knowledge of initial conditions of a chaotic process, no matter how small, will eventually be magnified until all knowledge is lost about the actual state of the system. Yet since the process is governed by a set of deterministic laws, the behavior of a chaotics system contains order.

A system modeled by Duffing's equation ($\ddot{x} + k\dot{x} + x^3 = B\ cos\,t$) (18) is a common example of a chaotic process. Looking at a Poincare section (figure 26) of the corresponding flow (depicted in the phase portrait below), it is clear that although this process is unpredictable, it contains a high degree of order.

THE LOGISTIC MAP

To illustrate the simplest example of chaos, consider the equation (23, 6):

$$X_{i+1} = \mu X_i(1 - X_i)$$

where μ is some parameter constant between 1 and 3. Following a 45 degree line to the intersection of the curve, we find the solution where $X_{i+1} = X_i$ (see figure 27). This point is a stable fixed point. As shown in the figure, iterations from any point (excluding 0 and 1) converge onto the fixed point. The fixed point represents an

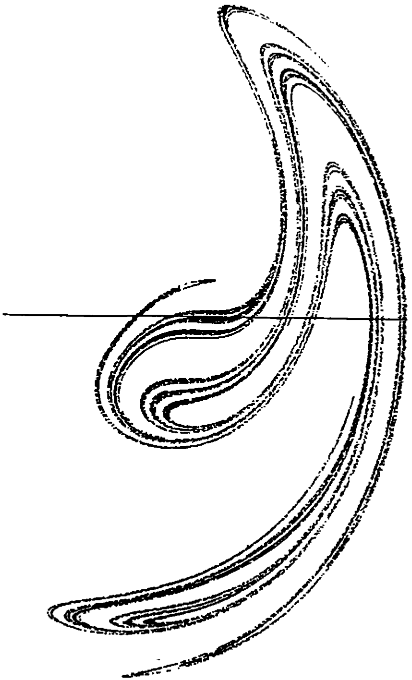

Figure 26. A Ueda attractor: An attractor of Duffing's equation

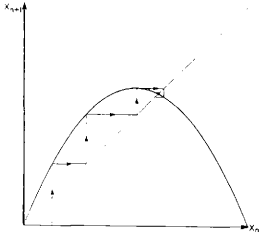

Figure 27. The logistic map for μ =2. 5. From (18)

equilibrium position; the slope at that point is less than 1 (45 degrees) and therefore paths in that vicinity tend to be drawn there.

If the parameter μ is increased to 3.1, fixed points emerge at four places: at zero which is still unstable (slope greater than 1), a new fixed stable point, the old fixed point which is now unstable, and a second new fixed stable point. Following figure 28, iterations no longer converge at the old fixed point but rather converge on a cycle between two new points. This is a simple example of a bifurcation since the old fixed point yielded two new fixed points. If μ is increased to 3.4, each of the fixed points becomes unstable and yields two more new stable fixed points.

At μ greater than 3.57, the equation becomes chaotic giving way to an infinite number of fixed points (see figure 29). With the exception of 0 and 1, all initial values of X will not converge onto any point at all, following a totally aperiodic path.

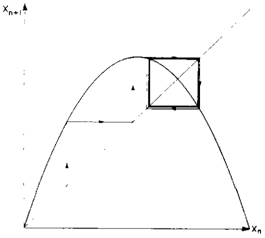

Figure 28. The logistic map for μ =3. 1. From (18)

DIMENSION OF ATTRACTORS

Most simply defined, the dimension of an attractor is a measure of complexity of the attractor. Essentially, the dimension is a lower bound on the number of variables required to describe the motion on the attractor. A fixed point, for instance, requires no variables to describe its behavior; therefore, the dimension of a fixed point is 0. A limit cycle is a simple curve and requires one variable to describe the location of the system along the closed orbit. However, the dimension of a chaotic attractor such as Ueda's attractor (figure 26) is not so straight-forward.

There are a variety of different approaches to calculate dimension; however, since each dimension is defined differently, they do not all measure the same thing. For instance, topological dimension is the quantity we are all familiar with which strictly

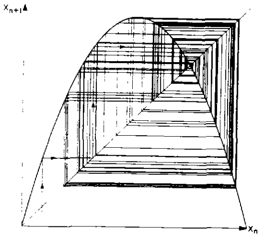

Figure 29. The logistic map for μ =3. 8. From (18)

measures the number of integer directions in a Euclidean space required to describe the coordinates of a set of points. The fractal dimension is basically a measure of the density of points within an attractor such that a line is of dimension one, while a plane is of dimension two. An attractor which consists of a set of points within a plane will have a dimension between one and two. The mathematical definition of fractal dimension, also called Hausdorff dimension is:

$$D_f = lim\ as\ \epsilon \rightarrow 0\ (\frac{lnN(\epsilon)}{ln(1/\epsilon)})$$

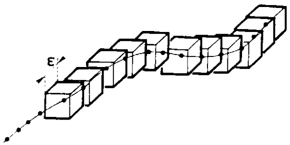

Figure 30. How the Hausdorff or fractal dimension is calculated. From (8)

where N represents the number of volume elements (hypercubes) required to cover the set of points on the attractor and ϵ is the length of the hypercube (see figure 30).

Another common dimension is the information dimension. Unlike the fractal dimension, which is a metric measure, information dimension measures the relative frequency of which a typical trajectory is visited.

$$D_i = lim \ as \ \epsilon \rightarrow 0 \ (\frac{ln H(\epsilon)}{ln(1/\epsilon)})$$

where $H = \sum_{i=1}^{N(\epsilon)} p_i \, ln \, p_i$. P_i is the relative probability a trajectory enters the i^{th} volume element.

Theoretically, a non-chaotic attractor will be of integer dimension while the opposite is true for a chaotic attractor. However, since dimension calculating algorithms are not very accurate, dimensions are useless to classify whether or not an attractor is chaotic. For instance, if the dimension of an attractor is 2.07, it is difficult to determine whether the attractor is chaotic or whether the deviation from an integer quantity is due to inaccuracy of the algorithm. Also, dimensions reveal nothing about the entire structure of an attractor. For the most part, dimensions are a general measure of complexity.

VITA

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